

## **HEAT AND MASS TRANSFER IN MHD FLOW OF A VISCOUS FLUID PAST A VERTICAL PLATE UNDER OSCILLATORY SUCTION VELOCITY WITH THERMAL RADIATION.**

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### **Abstract**

The present study deals with the effect of thermal radiation on Hydro magnetic heat and mass transfer in MHD flow of an incompressible, electrically conducting, viscous fluid past an infinite vertical porous plate embedded with porous medium of time dependent permeability under oscillatory suction velocity normal to the plate. It is considered that the influence of the uniform magnetic field acts normal to the flow and the permeability of the porous medium fluctuates with time. Solutions for the velocity field, temperature distribution and concentration distribution are obtained using Galerkin finite element method. Expressions for skin-friction, Nusselt number and Sherwood number are also obtained. The results obtained are discussed for Grashof number ( $Gr > 0$ ) corresponding to the cooling of the plate and ( $Gr < 0$ ) corresponding to the heating of the plate with the help of graphs and tables to observe the effects of various parameters encountered in the problem under investigation.

**Keywords:** Heat and Mass Transfer, MHD flow, Suction velocity, viscous fluid, Thermal Radiation, Galerkin finite element method.

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## 1. INTRODUCTION:

For some industrial applications such as glass production and furnace design in space technology applications, cosmlal flight aerodynamics rocket, propulsion systems, plasma physics which operate at higher temperatures, radiation effects can be significant. The phenomenon of heat and mass transfer has been the object of extensive research due to its applications in science and technology. Such phenomenon is observed in buoyancy induced motions in the atmosphere, in bodies of water, quasi-solid bodies, such as earth and so on. Unsteady oscillatory free convective flows play an important role in chemical engineering, turbo machinery and aerospace technology. Such flows arise due to either unsteady motion of a boundary or boundary temperature. Besides, unsteadiness may also be due to oscillatory free stream velocity or temperature. In the nature and industrial applications many transport processes exist in which heat and mass transfer takes place simultaneously as a result of combined buoyancy effect of thermal diffusion and diffusion of chemical species. In addition, the phenomenon of heat and mass transfer is also encountered in chemical process industries such as polymer production and food processing.

In recent years, progress has been considerably made in the study of free convection and mass transfer flow of a viscous fluid through porous medium. In these studies, the permeability of the porous medium is assumed to be constant while the porosity of the medium may not necessarily be constant because the porous material containing the fluid is a non-homogeneous medium. In the light of this fact, Srikanth *et al.*<sup>8</sup>, have investigated the effects of permeability variation on free convection flow past a vertical porous wall in a porous medium when the permeability varies in time. Singh *et al.*<sup>5</sup> have discussed hydromagnetic free convective and mass transfer flow of a viscous stratified fluid considering variation in permeability with direction. M. Acharya, *et al*<sup>3</sup> discussed magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. A. Kumar, *et al*<sup>1</sup> studied an unsteady oscillatory laminar free convection flow of an electrically conducting fluid through a porous medium along a porous hot vertical plate with time dependent suction in the presence of heat source/sink. Ganesh P, *et al*<sup>2</sup> studied finite difference analysis of unsteady natural convection MHD flow past an inclined plate with variable surface heat and mass flux. Md. Abdur Sattar, *et al*<sup>4</sup> studied The Effects of Variable properties and Hall current on steady MHD laminar convective fluid flow due to a porous rotating disk. Ogulu A, *et al.*<sup>6</sup> Unsteady

MHD free Convective flow of a compressible fluid past a moving vertical plate in the presence of Radiative heat transfer. Sk. A. Samad, *et al.*<sup>7</sup> MHD boundary layer flow over a heated stretching sheet with variable Viscosity. Srinivasa Rao.V and Anand Babu.L<sup>9</sup>, Studied Finite Element Analysis of Radiation And Mass Transfer Flow Past Semi- Infinite Moving Vertical Plate With Viscous Dissipation. Srinivasa Rao.V and Anand Babu.L<sup>10</sup>, discussed Finite Element Solution of MHD Free Convection Flow of an Incompressible Viscous Dissipative Fluid in an Infinite Vertical Oscillating Plate with Constant Heat Flux. In most of studies mentioned above, the oscillatory suction velocity in presence of time dependent viscosity along with the influence of uniform magnetic field and thermal radiation are not studied, while such flows are encountered in geophysical problems, astrophysical problems, soil sciences and so on. Therefore, the aim of the present investigation is to study the effects of permeability variation and oscillatory suction velocity on free convection and mass transfer flow of a viscous fluid past an infinite vertical porous plate to a porous medium when the plate is subjected to a time dependent suction velocity normal to the plate in the presence of a uniform transverse magnetic field and thermal radiation. The permeability of the porous medium is considered to be  $K_o(t') = K_o'(1 + \varepsilon e^{int'})$  and the suction velocity is assumed to be  $v(t') = -v_o(1 + \varepsilon e^{int'})$  where  $v_o > 0$  and  $\varepsilon \ll 1$  is a positive constant. The results of the study are discussed for various numerical values of the parameters.

## 2. MATHEMATICAL FORMULATION:

Consider unsteady hydro magnetic flow of an incompressible, electrically conducting, viscous fluid past an infinite vertical porous plate in a porous medium of time dependent permeability and suction velocity. In cartesian coordinate system, let x'-axis be along plate in the direction of the flow and y'-axis normal to it. A uniform magnetic field is introduced normal to the direction of the flow. In the analysis, we assumed that the magnetic Reynold's number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. Further, all the fluid properties are assumed to be constant except that of the influence of the density variation with temperature. Therefore, the basic flow in the medium is entirely due to buoyancy force caused by temperature difference between the wall and medium. Initially  $t' \leq 0$ , the plate as well as fluid is assumed to be at

the same temperature and the concentration of species is very low so that the Soret and Dofour effect are neglected. (Gebhart and Pera<sup>5</sup>). When  $t' > 0$ , the temperature of the plate is instantaneously raised (or lowered) to  $T_w'$  and the concentration of the species is raised (or lowered) to  $C_w'$ .

Under the stated assumptions and taking the usual Boussinesq's approximation in to account, the governing equations for momentum, energy and concentration in dimensionless form are:

$$\frac{\partial v'}{\partial t'} = 0 \quad (1)$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = g\beta(T' - T_\infty') + g\beta^*(C' - C_\infty') + \nu \frac{\partial^2 u'}{\partial y'^2} - \nu \frac{u'}{K_o'} - \frac{\sigma B_0^2 u'}{\rho} \quad (2)$$

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma T_\infty'^3}{3k_e \rho C_p} \frac{\partial^2 T'}{\partial y'^2} \quad (3)$$

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} \quad (4)$$

where  $u'$  is the velocity along the x-axis,  $\nu'$  is the kinematic coefficient of viscosity,  $g$  is the acceleration due to gravity,  $\beta$  is the coefficient of volume expansion for the heat transfer,  $\beta^*$  is the volumetric coefficient of expansion with species concentration,  $T'$  is the fluid temperature,  $T_\infty'$  is the fluid temperature at infinity,  $C'$  is the species concentration,  $C_\infty'$  is the species concentration at infinity,  $D$  is the chemical molecular diffusivity,  $\varepsilon$  - porosity of the porous medium,  $k$  - Mean absorption co-efficient,  $K_o'$  is the constant permeability of the medium,  $\sigma$  is the scalar electrical conductivity,  $\mu$  is the coefficient of viscosity,  $C_p$  is the specific heat at constant pressure,  $\eta$  is the frequency of oscillation,  $\rho$  is the density of the fluid and  $t$  is the time.

The corresponding boundary conditions are

$$\left. \begin{aligned} u' = 0, \quad T' = T'_w + \varepsilon(T'_w - T'_\infty)e^{in't'} \quad C' = C'_w + \varepsilon(C'_w - C'_\infty)e^{in't'} \quad \text{at } y' = 0 \\ u' \rightarrow 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \quad \text{as } y' \rightarrow \infty \end{aligned} \right\} \quad (5)$$

From the continuity equation, it can be seen that  $v'$  is either a constant or a function of time. So assuming suction velocity to be oscillatory about a non-zero constant mean, one can write  $v' = -v_0(1 + \varepsilon e^{in't'})$  where  $v_0$  is the mean suction velocity,  $n$  is frequency of oscillation and  $v_0 > 0$ ,  $\varepsilon \ll 1$  is a positive constant. The negative sign indicates that the suction velocity is directed towards the plate. The permeability of the porous medium is considered to be  $K_o(t') = K'_o(1 + \varepsilon e^{in't'})$

The non dimensionless quantities introduced in these equations are defined as:

$$\begin{aligned} y &= \frac{v_0 y'}{4v}; \quad t = \frac{v_0^2 t'}{4v}; \quad n = \frac{4vn'}{v_0^2}; \quad u = \frac{u'}{v_0}; \quad T = \frac{T' - T_\infty}{T'_w - T_\infty}; \quad C = \frac{C' - C_\infty}{C'_w - C_\infty}; \\ K_o &= \frac{K'_o v_0^2}{v^2} (\text{Constant permeability of the medium}); \\ Gr &= \frac{vg\beta^*(T'_w - T_\infty)}{v_0^3} (\text{Grashof Number}); \quad Sc = \frac{\nu}{D} (\text{Schmidt Number}); \\ Pr &= \frac{\mu C_p}{K_t} (\text{Prandtl Number}); \quad M = \frac{B_o}{v_0} \sqrt{\frac{\sigma \nu}{\rho}} (\text{Magnetic Parameter}) \\ Gm &= \frac{vg\beta(C'_w - C_\infty)}{v_0^3} (\text{Modified Grashof Number}); \quad R = \frac{kk_e}{4\sigma^* T_\infty^3} (\text{Thermal Radiation}); \end{aligned}$$

The governing equations for momentum, energy and concentration in dimensionless form are:

$$\frac{1}{4} \frac{\partial u}{\partial t} - \left(1 + \varepsilon e^{int}\right) \frac{\partial u}{\partial y} = Gr T + Gm C + \frac{\partial^2 u}{\partial y^2} - \frac{u}{K_o(1 + \varepsilon e^{int})} - M^2 u \quad (6)$$

$$\frac{1}{4} \frac{\partial T}{\partial t} - (1 + \varepsilon^{\text{int}}) \frac{\partial T}{\partial y} = \frac{1}{\text{Pr}} \left( 1 + \frac{4}{3R} \right) \frac{\partial^2 T}{\partial y^2} \quad (7)$$

$$\frac{1}{4} \frac{\partial C}{\partial t} - (1 + \varepsilon^{\text{int}}) \frac{\partial C}{\partial y} = \frac{1}{\text{Sc}} \frac{\partial^2 C}{\partial y^2} \quad (8)$$

The relevant boundary conditions in dimensionless form are

$$\left. \begin{aligned} u = 0, \quad T = 1 + \varepsilon^{\text{int}}, \quad C = 1 + \varepsilon^{\text{int}} \quad \text{at } y = 0 \\ u \rightarrow 0, T \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (9)$$

### 3. METHOD OF SOLUTION:

By applying Galerkin finite element method for equation (6) over the element (e),

( $y_i \leq y \leq y_k$ ) is:

$$\int_{y_j}^{y_k} \left\{ N^T \left[ \frac{\partial^2 u^{(e)}}{\partial y^2} - \frac{1}{4} \frac{\partial u^{(e)}}{\partial t} + A \frac{\partial u^{(e)}}{\partial y} - Ru^{(e)} + P \right] \right\} dy = 0 \quad (10)$$

Where  $A = 1 + \varepsilon e^{\text{int}}$ ,  $R = \frac{1}{K_o A} + M^2$ ,  $P = (Gr)T + (Gm)C$

Integrating the first term in Eq. (10) by parts one obtains

$$N^{(e)T} \left\{ \frac{\partial u^{(e)}}{\partial y} \right\}_{y_j}^{y_k} - \int_{y_j}^{y_k} \left\{ \frac{\partial N^{(e)T}}{\partial y} \frac{\partial u^{(e)}}{\partial y} + N^{(e)T} \left( \frac{1}{4} \frac{\partial u^{(e)}}{\partial t} - A \frac{\partial u^{(e)}}{\partial y} + Ru^{(e)} - P \right) \right\} dy = 0 \quad (11)$$

Neglecting the first term in Eq. (11), one gets:

$$\int_{y_j}^{y_k} \left\{ \frac{\partial N^{(e)T}}{\partial y} \frac{\partial u^{(e)}}{\partial y} + N^{(e)T} \left( \frac{1}{4} \frac{\partial u^{(e)}}{\partial t} - A \frac{\partial u^{(e)}}{\partial y} + Ru^{(e)} - P \right) \right\} dy = 0$$

Let  $u^{(e)} = N^{(e)} \phi^{(e)}$  be the linear piecewise approximation solution over the element (e)

( $y_i \leq y \leq y_k$ ), where  $N^{(e)} = [N_j \quad N_k]$ ,  $\phi^{(e)} = [u_j \quad u_k]^T$  and  $N_j = \frac{y_k - y}{y_k - y_j}$ ,

$N_k = \frac{y - y_j}{y_k - y_j}$  are the basis functions. One obtains:

$$\int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j' & N_j' & N_j' & N_k' \\ N_j' & N_k' & N_k' & N_k' \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy + \frac{1}{4} \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j & N_j & N_j & N_k \\ N_j & N_k & N_k & N_k \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} \right\} dy$$

$$- \frac{A}{2l^{(e)}} \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j & N_j' & N_j & N_k' \\ N_j' & N_k & N_k' & N_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy + \frac{R}{6} \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j & N_j & N_j & N_k \\ N_j & N_k & N_k & N_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy = P \int_{y_j}^{y_k} \begin{bmatrix} N_j \\ N_k \end{bmatrix} dy$$

Simplifying we get

$$\frac{1}{l^{(e)^2}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{1}{24} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} - \frac{A}{2l^{(e)}} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{R}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = \frac{P}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Where prime and dot denotes differentiation w.r.to 'y' and time 't' respectively. Assembling the element equations for two consecutive elements  $y_{i-1} \leq y \leq y_i$  and  $y_i \leq y \leq y_{i+1}$  following is obtained:

$$\frac{1}{l^{(e)^2}} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{1}{24} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_{i-1} \\ \dot{u}_i \\ \dot{u}_{i+1} \end{bmatrix} - \frac{A}{2l^{(e)}} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix}$$

$$+ \frac{R}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} = \frac{P}{2} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad (12)$$

Now put row corresponding to the node 'i' to zero, from Eq. (12) the difference schemes with  $l^{(e)} = h$  is:

$$\frac{1}{h^2}[-u_{i-1} + 2u_i - u_{i+1}] + \frac{1}{24}\left[\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1}\right] - \frac{A}{2h}[-u_{i-1} + u_{i+1}] + \frac{R}{6}[u_{i-1} + 4u_i + u_{i+1}] = P \quad (13)$$

Applying the trapezoidal rule, following system of equations in Crank-Nicholson method are obtained:

$$A_1 u_{i-1}^{n+1} + A_2 u_i^{n+1} + A_3 u_{i+1}^{n+1} = A_4 u_{i-1}^n + A_5 u_i^n + A_6 u_{i+1}^n + 12Phk \quad (14)$$

Now from Eqs (7) and (8), following equations are obtained:

$$B_1 T_{i-1}^{n+1} + B_2 T_i^{n+1} + B_3 T_{i+1}^{n+1} = B_4 T_{i-1}^n + B_5 T_i^n + B_6 T_{i+1}^n \quad (15)$$

$$C_1 C_{i-1}^{n+1} + C_2 C_i^{n+1} + C_3 C_{i+1}^{n+1} = C_4 C_{i-1}^n + C_5 C_i^n + C_6 C_{i+1}^n \quad (16)$$

where  $A_1 = h + 2Rk - 12rh + 6Ak$ ;  $A_2 = 4h + 24rh + 8Rkh$ ;  $A_3 = h + 2Rk - 12rh - 6Ak$ ;

$A_4 = h - 2Rk + 12rh - 6Ak$ ;  $A_5 = 4h - 24rh - 8Rkh$ ;  $A_6 = h - 2Rk + 12rh + 6Ak$ ;

$P = 24h(Gr)kT_i^j + 24h(Gm)kC_i^j$ ;  $B_1 = 1+6Ahr-12Br$ ;  $B_2 = 4+24Br$ ;

$B_3 = 1-12Br-6Ahr$ ;  $B_4 = 1-6Ahr+12Br$ ;  $B_5 = 4-24Br$ ;  $B_6 = 1+12Br+6Ahr$ ; and  $B =$

$$\frac{1}{Pr}\left(1 + \frac{4}{3R}\right)$$

$C_1 = h(Sc) + 6Ak(Sc) - 12rh$ ;  $C_2 = 4h(Sc) + 24rh$ ;  $C_3 = h(Sc) - 6Ak(Sc) - 12rh$ ;

$C_4 = h(Sc) - 6Ak(Sc) + 12rh$ ;  $C_5 = 4h(Sc) - 24rh$ ;  $C_6 = h(Sc) + 6Ak(Sc) + 12rh$ ;

Here  $r = \frac{k}{h^2}$  and  $h, k$  are mesh sizes along  $y$ - direction and time-direction respectively.

Index 'i' refers to space and 'j' refers to the time. In the equations (14), (15) and (16), taking  $i = 1(1) n$  and using boundary conditions (9), then the following system of equations are obtained:

$$A_i X_i = B_i \quad i = 1(1)3 \quad (17)$$

Where  $A_i$ 's are matrices of order  $n$  and  $X_i, B_i$ 's are column matrices having  $n$ -components. The solutions of above system of equations are obtained by using Thomas algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by C-programme. In order to prove the convergence and stability of Galerkin finite element method, the same C-programme was run with smaller values of  $h$  and  $k$  and no significant change was observed in the values of  $u$ ,  $T$  and  $C$ . Hence the Galerkin finite element method is stable and convergent.

### SKIN-FRICTION, RATE OF HEAT AND MASS TRANSFER:

Skin-Friction co-efficient ( $\tau$ ) at the plate is 
$$\tau = \left( \frac{\partial u}{\partial y} \right)_{y=0}$$

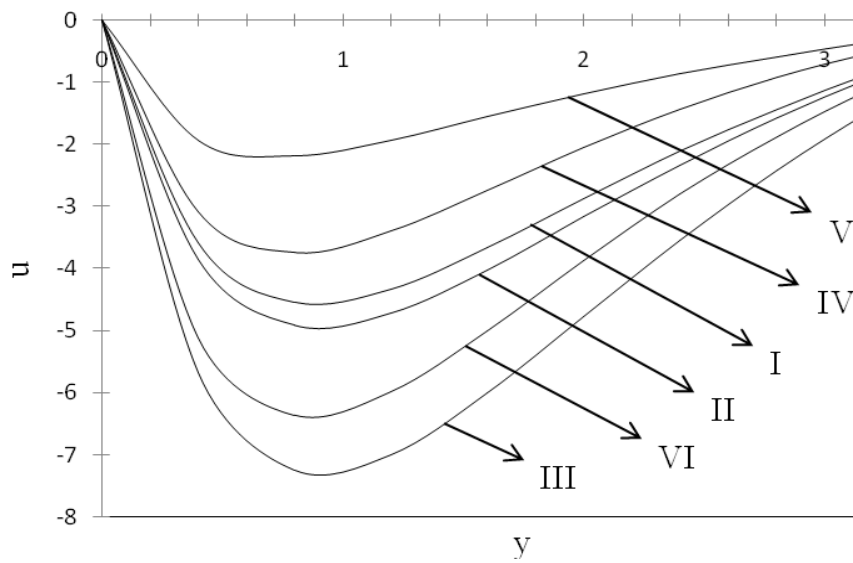
Heat transfer co-efficient ( $N_u$ ) at the plate is 
$$N_u = - \left( \frac{\partial \theta}{\partial y} \right)_{y=0}$$

Mass transfer co-efficient ( $S_b$ ) at the plate is 
$$S_b = - \left( \frac{\partial C}{\partial y} \right)_{y=0}$$

### RESULTS AND DISCUSSIONS:

In order to get physical insight into the problem velocity field, temperature and concentration field, skin-friction coefficient, rate of heat and mass transfer have been discussed by assigning numerical values to various parameters appeared in the equations obtained in mathematical formulation in the problem. To be realistic, the values of the Prandtl number are chosen for mercury ( $Pr = 0.025$ ), air ( $Pr = 0.71$ ), water ( $Pr = 7.0$ ) and water at  $4^\circ\text{C}$  ( $Pr = 11.40$ ). The values of Schmidt number are taken for Hydrogen ( $Sc = 0.22$ ), Helium ( $Sc = 0.30$ ), Water vapour ( $Sc = 0.60$ ), Oxygen ( $Sc = 0.66$ ) and Ammonia ( $Sc = 0.78$ ). Two cases of general interest for Grashof number  $Gr > 0$  corresponding to cooling of the plate and Grashof number  $Gr < 0$  corresponding to heating of the plate are considered. For the figures of velocity profiles, the numerical values of Schmidt number are chosen to be  $Sc = 0.22, 0.66$ , magnetic parameter  $M = 0.5, 1.0$  and permeability parameter  $Ko = 10.0, 20.0$  at frequency parameter  $n = 5.0$ , Prandtl number  $Pr = 0.71$ , perturbation parameter  $\varepsilon = 0.005$  and

$nt = \pi/2$ . The values of Grashof number for heat transfer are chosen to be  $Gr = 10.0, 20.0$  and the values of modified Grashof number for mass transfer are taken as  $Gm = 10.0, 20.0$  corresponding to cooling of the plate while corresponding to heating of the plate the numerical values of Grashof number for heat transfer are chosen to be  $Gr = -10.0, -20.0$ . For the tables, the values of  $Gr$  are chosen to be  $10.0, 20.0$  and the values of  $Gm$  are chosen to be  $4.0$  and  $8.0$  for cooling of the plate while these values of  $Gr$  are chosen to be  $-10.0, -20.0$  and the values of  $Gm$  are chosen to be  $-4.0$  and  $-8.0$  for heating of the plate. The values of other parameters are taken as considered in figures.

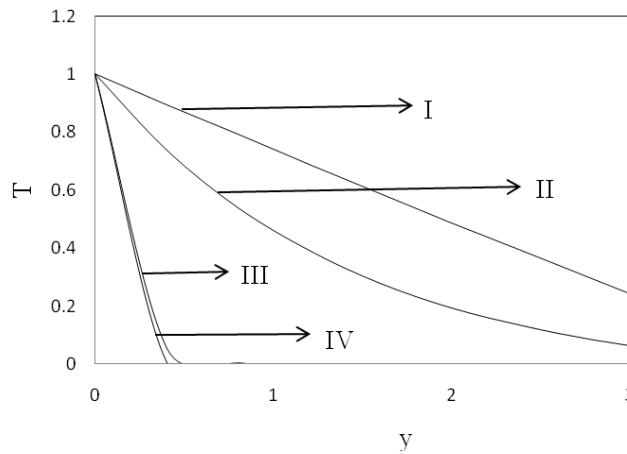


| Curve | Gr   | Gm   | Sc   | M   | $K_o$ |
|-------|------|------|------|-----|-------|
| I     | 10.0 | 10.0 | 0.22 | 0.5 | 10.0  |
| II    | 20.0 | 10.0 | 0.22 | 0.5 | 10.0  |
| III   | 10.0 | 20.0 | 0.22 | 0.5 | 10.0  |
| IV    | 10.0 | 10.0 | 0.66 | 0.5 | 10.0  |
| V     | 10.0 | 10.0 | 0.22 | 1.0 | 10.0  |
| VI    | 10.0 | 10.0 | 0.22 | 0.5 | 20.0  |

**Fig. 1. Transient velocity field due to cooling of the plate ( $n = 5.0$ ,  $\varepsilon = 0.005$ ,  $Pr = 0.71$ ,  $nt = \pi/2$ )**

Fig. 1 shows transient velocity field due to cooling of the plate as a result of change in the numerical values of Grashof number for heat transfer  $Gr$ , modified Grashof number for mass transfer  $Gm$ , Schmidt number  $Sc$ , magnetic parameter  $M$  and permeability parameter  $K_0$  at  $\varepsilon = 0.002$ ,  $Pr = 0.71$  and  $nt = \pi/2$ . It is observed that an increase in  $Gr$  or  $Gm$  or  $K_0$  decreases the transient velocity field while an increase in  $Sc$  or  $M$  increases the transient velocity field. A comparison of transient velocity distribution curves due to cooling of the plate show that in the vicinity of the plate the velocity falls very rapidly and thereafter steadily indicating that the curves rise gradually after attaining minimum value near the plate.

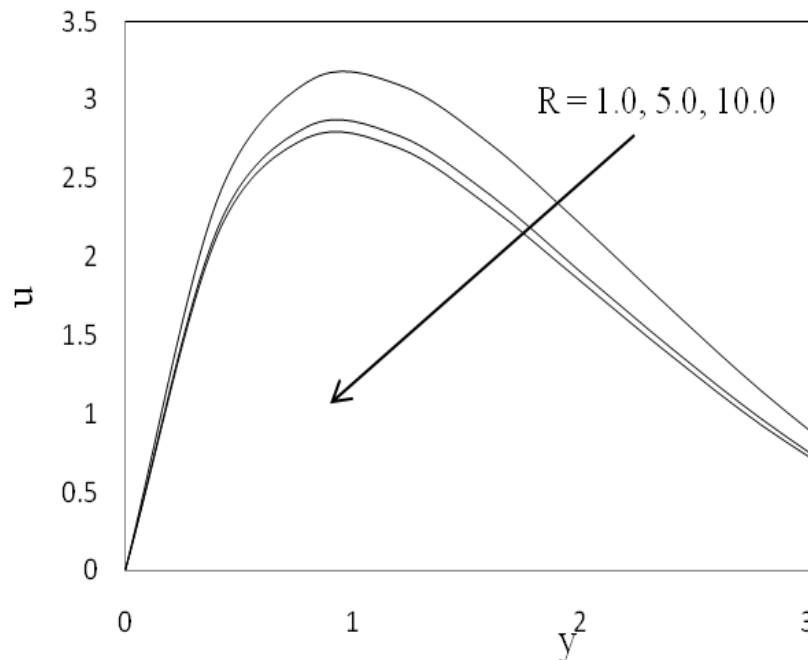
An important remark to be mentioned here is that for heating of the plate ( $Gr < 0$ ), an interesting flow phenomenon was observed on the screen of the computer when negative sign was introduced before the numerical values of  $Gr$  and  $Gm$  for the same numerical values of the Fig. 1. It was observed that the numerical values of the velocity remained the same but opposite in sign. This indicates that if we draw figure for heating case it will be similar to figure-1 except for the y-coordinates of figure-1 which are negative in cooling case. These y-coordinates will be positive for heating case.



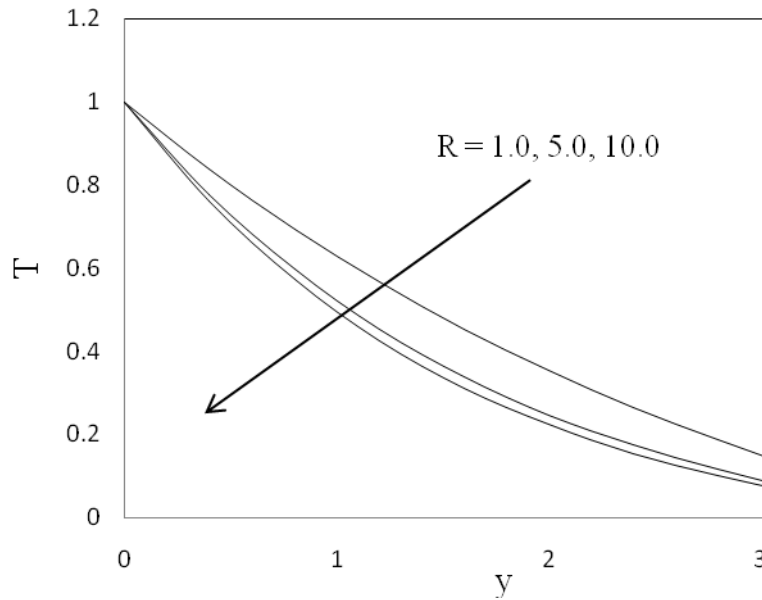
|     |                            |              |
|-----|----------------------------|--------------|
| I   | Mercury                    | $Pr = 0.025$ |
| II  | Air                        | $Pr = 0.71$  |
| III | Water                      | $Pr = 7.00$  |
| IV  | Water at $4^\circ\text{C}$ | $Pr = 11.40$ |

**Fig. 2. Transient temperature due to variation in Prandtl number ( $n = 5.0$ ,  $\varepsilon = 0.005$ ,  $nt = \pi/2$ )**

Fig. 2 shows transient temperature field due to variation in Prandtl number  $Pr$  for air, mercury, water and water at  $4^\circ\text{C}$  at  $\varepsilon = 0.002$  and  $nt = \pi/2$ . It is observed that an increase in  $Pr$  decreases the transient temperature field indicating that temperature field falls more rapidly for water in comparison to air. An important observation noted from the figure is that the temperature field remains almost stationary for mercury which is most sensible towards change in temperature. This leads to the conclusion that mercury is most effective for maintaining temperature differences and can be efficiently used in laboratory purposes. It is the air which may replace mercury but the effectiveness of maintaining temperature changes is much less than mercury. However, air can be a better and cheap replacement if the temperature is maintained for industrial purposes.



**Fig. 3.** Effect of Thermal Radiation parameter ' $R$ ' on velocity profiles ' $u$ ' when  $Gr = 5.0$ ,  $Gm = 5.0$ ,  $Sc = 0.22$ ,  $Pr = 0.71$ ,  $M = 1.0$  and  $nt = \pi/2$ .



**Fig. 4. Effect of Thermal Radiation parameter ‘R’ on temperature profiles ‘T’ when Gr = 5.0, Gm = 5.0, Sc = 0.22, Pr = 0.71, M = 1.0, and  $nt = \pi/2$ .**

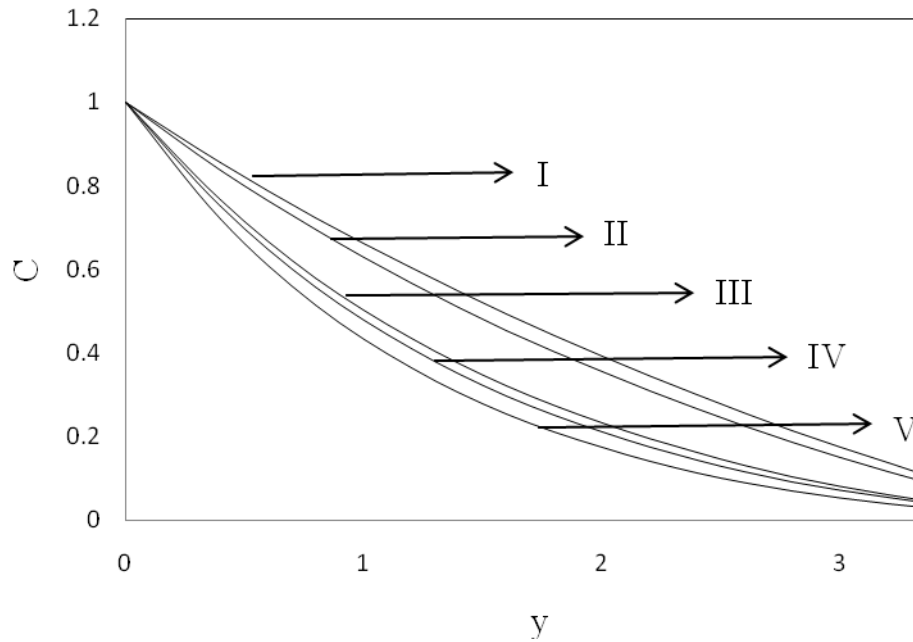
Fig. (3) and (4) shows the velocity and temperature profiles for different values of the thermal Radiation parameter  $R$ .  $R = \frac{k_e k}{4\sigma^* T_\infty^3}$  and this defines the ratio of thermal conduction

contribution relative to the thermal radiation. For radiative heat transfer dominance in the boundary layer regime,  $R \rightarrow 0$ . For finite values of  $R$  there will be a simultaneous presence of thermal conduction and radiative transfer contribution. For  $R=1$  both modes will contribute equally. For  $R \rightarrow \infty$ , in equation (7), the term  $4/3R \rightarrow 0$  and the energy conservation equation reduces to the convectational unsteady conduction-convection equation i.e.

$$\frac{1}{4} \frac{\partial T}{\partial t} - (1 + \epsilon^{\text{int}}) \frac{\partial T}{\partial y} = \frac{1}{\text{Pr}} \frac{\partial^2 T}{\partial y^2}$$

An increase in  $R$  from 0 (total thermal radiation dominance) through 1.0, 5.0 to 10.0, causes a significant decrease in velocity with distance into the boundary layer i.e. decelerates the flow. Velocities in all cases ascend from the plate surface, peak close to the wall and then decay smoothly to zero in the free stream. Thermal radiation flux therefore has a destabilizing effect on the transient flow regime. This is important in polymeric and other

industrial flow processes since it shows that the presence of thermal radiation while decreasing temperature, will affect flow control from the plate surface into the boundary layer regime. As expected, temperature values are also significantly reduced with an increase in  $R$  as there is a progressive decrease in thermal radiation contribution accompanying this. All profiles are monotonic decay from the wall to the free stream.



|     |              |             |
|-----|--------------|-------------|
| I   | Hydrogen     | $Sc = 0.22$ |
| II  | Helium       | $Sc = 0.30$ |
| III | Water-vapour | $Sc = 0.60$ |
| IV  | Oxygen       | $Sc = 0.66$ |
| V   | Ammonia      | $Sc = 0.78$ |

Fig. 3. Transient concentration field due to variation in Schmidt number ( $n = 5.0$ ,  $\varepsilon = 0.005$ ,  $nt = \pi/2$ )

Fig. 3 shows transient concentration field due to variation in Schmidt number  $Sc$  for the gases Hydrogen, Helium, Water-vapour, Oxygen and Ammonia at  $\varepsilon = 0.002$  and  $nt = \pi/2$ . It is observed that concentration field falls slowly and steadily for Hydrogen and Helium but

falls rapidly for Oxygen and Ammonia in comparison to Water-vapour. Thus Hydrogen can be used for maintaining effective concentration field and Water-vapour can be used for maintaining normal concentration field.

| Gr   | Gm  | M   | Sc   | Pr   | Ko   | R   | $\tau$  |
|------|-----|-----|------|------|------|-----|---------|
| 10.0 | 4.0 | 0.5 | 0.22 | 0.71 | 10.0 | 1.0 | 09.3769 |
| 20.0 | 4.0 | 0.5 | 0.22 | 0.71 | 10.0 | 1.0 | 15.3029 |
| 10.0 | 8.0 | 0.5 | 0.22 | 0.71 | 10.0 | 1.0 | 12.8276 |
| 10.0 | 4.0 | 0.5 | 0.66 | 0.71 | 10.0 | 1.0 | 08.3830 |
| 10.0 | 4.0 | 1.0 | 0.22 | 0.71 | 10.0 | 1.0 | 05.4383 |
| 10.0 | 4.0 | 0.5 | 0.22 | 0.71 | 20.0 | 1.0 | 10.0271 |
| 10.0 | 4.0 | 0.5 | 0.22 | 7.00 | 10.0 | 1.0 | 04.2190 |
| 10.0 | 4.0 | 0.5 | 0.22 | 0.71 | 10.0 | 5.0 | 08.5218 |

**Table 1. Skin-Friction co-efficient of ( $\tau$ ) for cooling of the plate.**

Table 1. represents the numerical values of skin-friction coefficient ( $\tau$ ) for variations in Gr, Gm, M, Sc, Pr, and Ko respectively, corresponding to cooling of the plate. An increase in Gr or Gm or Ko leads to an increase in the value of skin-friction co-efficient while in increase in M or Sc or Pr or R leads to a decrease in the value of skin-friction coefficient.

| Gr    | Gm  | M   | Sc   | Pr   | Ko   | R   | $\tau$  |
|-------|-----|-----|------|------|------|-----|---------|
| -10.0 | 4.0 | 0.5 | 0.22 | 0.71 | 10.0 | 1.0 | -2.4753 |
| -20.0 | 4.0 | 0.5 | 0.22 | 0.71 | 10.0 | 1.0 | -8.4013 |
| -10.0 | 8.0 | 0.5 | 0.22 | 0.71 | 10.0 | 1.0 | 0.9755  |
| -10.0 | 4.0 | 0.5 | 0.66 | 0.71 | 10.0 | 1.0 | -3.4691 |
| -10.0 | 4.0 | 1.0 | 0.22 | 0.71 | 10.0 | 1.0 | -1.7868 |
| -10.0 | 4.0 | 0.5 | 0.22 | 0.71 | 20.0 | 1.0 | -2.5398 |
| -10.0 | 4.0 | 0.5 | 0.22 | 7.00 | 10.0 | 1.0 | 2.6826  |
| -10.0 | 4.0 | 0.5 | 0.22 | 0.71 | 10.0 | 5.0 | 2.1645  |

**Table 2. Skin-Friction co-efficient of ( $\tau$ ) for heating of the plate.**

Table 2. represents the numerical values of skin-friction coefficient ( $\tau$ ) for variations in Gr, Gm, M, Sc, Pr, and Ko respectively, corresponding to heating of the plate. An increase in Gr or Gm or Pr or R or M leads to an increase in the value of skin-friction co-efficient while in increase in Sc or Ko leads to a decrease in the value of skin-friction coefficient.

| Pr     | R   | $N_u$  |
|--------|-----|--------|
| 00.025 | 0.5 | 0.1239 |
| 00.710 | 0.5 | 0.6868 |
| 07.000 | 0.5 | 5.1852 |
| 11.400 | 0.5 | 7.2611 |
| 00.025 | 1.0 | 0.0539 |

**Table 3. Heat transfer co-efficient in terms of Nusselt number.**

Table 3. represents the numerical values of heat transfer co-efficient ( $N_u$ ) for different values of Prandtl number Pr. An increase in Pr leads to an increase in heat transfer co-efficient.

Also the value of  $N_u$  is least for Mercury and highest for Water at  $4^\circ\text{C}$ . while an increase in the thermal radiation parameter leads to decrease in the value of heat transfer coefficient.

| Sc   | $S_b$  |
|------|--------|
| 0.22 | 0.2525 |
| 0.30 | 0.3168 |
| 0.60 | 0.5852 |
| 0.66 | 0.6406 |
| 0.78 | 0.7514 |
| 1.00 | 0.9525 |
| 2.62 | 2.3165 |

**Table 4. Mass transfer co-efficient in terms of Sherwood number.**

Table 4. represents the numerical values of mass transfer co-efficient ( $S_b$ ) for different values of Schmidt number Sc. An increase in Sc leads to an increase in mass transfer co-efficient. Also, the value of  $S_b$  is least for Hydrogen and highest for Propyl benzene.

### CONCLUSIONS:

The problem “heat and mass transfer in MHD Flow of a viscous fluid past a vertical plate under oscillatory suction velocity with thermal radiation” is studied. The dimensionless equations are solved by using Galerkin finite element method. The Effects of Velocity, Temperature and Concentration for different parameters like Gr, Gm, Pr, Sc, M, R, Ko,  $\varepsilon$  and  $nt$  are studied. The study concludes the following results.

- 1). The velocity decreases with the increasing of Magnetic parameter M, Schmidt number Sc, Radiation parameter R.
- 2). The velocity decreases with the increase of Prandtl number Pr for cooling of the plate ( $Gr < 0$ ) and the velocity increases with the increase of Prandtl number Pr for heating of the plate ( $Gr > 0$ ).
- 3). The velocity increases with the increasing of Grashof number Gr and Modified Grashof number Gm.
- 4). The temperature and Concentration decreases with increasing of Prandtl number Pr, Radiation parameter R and Schmidt number Sc respectively.

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